

EXPLORING PROBLEM-SOLVING SKILLS PROFILES OF PRE-SERVICE MATHEMATICS TEACHERS THROUGH POLYA'S FOUR STAGES

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Abstract

Mathematical problem-solving skills are an essential competence for pre-service mathematics teachers, as they are expected to cultivate these skills in their future students. This study aims to describe the problem-solving profiles of pre-service mathematics teachers using Polya's four stages as an analytical lens. A qualitative case study approach was employed to examine the cognitive processes demonstrated by pre-service teachers while solving mathematical problems. The findings reveal diverse problem-solving profiles, including conceptual understanding shaped by prior experience, logical reasoning used to interpret problem conditions, and the ability to represent problems through mathematical models. These profiles underscore the need for dedicated coursework that enhances mathematical problem-solving skills within mathematics education programs. The study provides insights into the variation of problem-solving profiles and offers implications for curriculum development in the mathematics education department.

Keywords: Mathematical problem-solving skills, pre-service mathematics teachers, Polya's profile, qualitative research

INTRODUCTION

Problem-solving is viewed as a reform in curriculum, which enables students to apply concepts and skills, engage with ideas, and develop new skills (Hiebert et al., 1996; Wahyuni et al., 2025; Widodo et al., 2025). These learning conditions can cultivate students' flexibility, creativity, and confidence in tackling unfamiliar problems, allowing mathematics to become a meaningful and intellectually engaging activity (Foster, 2023). In addition, problem-solving also fosters decision-making, logical reasoning, critical thinking, and creativity, which are essential for 21st-century students (Widjanti, 2009).

Teachers play a central role in nurturing students' problem-solving skills. They can assess students' competence by observing engagement and strategies during problem-solving tasks (Putri et al., 2022). Moreover, as educators and facilitators, teachers can guide students through structured problem-solving activities and reflective practices (Pradnyana & Amanda, 2023). Competent teachers in problem-

solving are therefore more effective in fostering these essential skills in their students (Schoenfeld, 2016).

Pre-service mathematics teachers must possess strong problem-solving skills themselves. These skills can be developed through targeted lectures, guided exercises, and active engagement with mathematical tasks (Widjanti, 2009). However, challenges in cultivating these skills include selecting problems appropriate for a diverse student population, as relevance may vary among individuals (Abuhasanein et al., 2025).

Additionally, several studies have highlighted the challenges faced by pre-service teachers in mathematical problem-solving. Yayuk & Husamah, (2020) analyzed the challenges encountered by elementary pre-service teachers in solving mathematical problems using Polya's steps and found that most students struggled with understanding the problem, planning the solution, executing the plan, and reviewing the results. Similarly, Berenger (2018) investigated how pre-service teachers approached structured problem-solving as part of their first-year mathematics education

courses, reporting that analysis of pre-service teachers' written work revealed notable gaps in their ability to clearly articulate their thinking processes and accurately apply mathematical concepts during problem solving.

To address these challenges, a structured theoretical framework is necessary. George Polya's problem-solving theory, like as understanding the problem, devising a plan, carrying out the plan, and looking at the results, provides a comprehensive approach to analyze and enhance mathematical problem-solving processes (Polya, 1985). This framework emphasizes systematic thinking by guiding learners to identify given information, select appropriate strategies, execute solutions coherently, and reflect on the correctness of their results. Previous studies have demonstrated that the application of Polya's stages supports the development of analytical skills and fosters informed decision-making through structured information processing and reflection (Schoenfeld, 2016). In mathematics education, particularly for pre-service teachers, Polya's framework has been widely adopted to examine variations in problem-solving strategies and to identify strengths and gaps across different stages of the problem-solving process (Kania & Fitriyani, 2022; Sumarni et al., 2021). Therefore, this framework is considered appropriate for investigating the problem-solving profiles of pre-service mathematics teachers.

Developing problem-solving competence in pre-service teachers carries significant implications for mathematics education. Teachers who possess strong problem-solving skills are better equipped to design effective learning experiences, foster similar abilities in their students, and adapt instructional strategies to diverse classroom contexts. These competencies are fundamental for preparing mathematics educators capable of responding to contemporary teaching challenges. Research has demonstrated that systematic interventions, such as structured pedagogical approaches and problem-based learning (PBL), can significantly enhance these skills in pre-service teachers (Ajani, 2024; Özlem

& Arzu, 2008). Specifically, Ajani, (2024) emphasizes that fostering problem-solving skills through targeted instructional strategies and reflective practices in higher education supports both conceptual understanding and practical application. Similarly, Özlem & Arzu (2008) found that PBL effectively strengthens pre-service teachers' problem-solving skills, enabling them to transfer these competencies into classroom teaching and promote problem-solving among their future students. Collectively, these findings underscore the crucial role of deliberate instructional design in developing the problem-solving capabilities of future mathematics educators.

Despite these promising strategies, research indicates that the development of problem-solving skills in pre-service teachers remains uneven. While some students demonstrate advanced conceptual understanding shaped by prior experience, others encounter difficulties in interpreting problems or have limited exposure to diverse problem types (Sumarni et al., 2021; Widjajanti, 2009) recognizing these differences is essential for tailoring instructional support and enhancing the overall effectiveness of teacher education programs.

Several studies have further underscored the importance of structured problem-solving frameworks in teacher education. For example, Polya's stages provide a systematic approach to improving students' analytical and modeling abilities, fostering reflective thinking, and facilitating application across various contexts (Polya, 1985; Widjajanti, 2009). Nevertheless, empirical evidence documenting the diverse profiles of problem-solving skills among pre-service mathematics teachers remains limited.

Understanding these profiles is crucial for curriculum development. By mapping students' problem-solving skills, educators can design targeted interventions, provide differentiated support, and ensure that graduates are equipped to cultivate problem-solving skills in their future classrooms (Santos-Trigo, 2024; Tractenberg et al., 2024). Such an approach aligns with the broader goal of producing competent, reflective, and adaptive mathematics teachers.

Moreover, identifying problem-solving profiles helps bridge the gap between theoretical knowledge and practical application in teacher education (Qohar, 2023; Umi et al., 2024). It

highlights areas of strength and aspects that require additional guidance, promoting a comprehensive approach to skill development.

Given the centrality of problem-solving in mathematics education and the necessity for effective teacher preparation, the present study aims to investigate the diverse profiles of mathematical problem-solving skills among pre-service mathematics teachers. Polya's framework serves as the analytical lens to examine how students approach and resolve mathematical problems.

Accordingly, this study addresses the following research question: *"What are the diverse profiles of mathematical problem-solving skills among pre-service mathematics teachers based on Polya's theoretical stages?"* The findings are expected to inform teacher education programs, curriculum design, and instructional strategies that aim to enhance problem-solving competence in future mathematics teachers.

METHOD

This study employed a qualitative case study approach to analyze the cognitive processes and strategies used by pre-service mathematics teachers in mathematical problem-solving. This approach was chosen because the study focused on describing observable problem-solving of the pre-service mathematics teachers based on Polya's stages. According to Creswell (2014), the qualitative approach is designed when researchers aim to provide straightforward, detailed descriptions of participants' actions, responses, and thought processes without requiring extensive interpretation.

This research was conducted at a private university in Pekanbaru, Riau Province. The participants in this study were sixth-semester students of mathematics

education. A total of 20 participants took part in this study. The research was conducted at the end of the odd semester of the 2024/2025 academic year.

The data collection instruments consisted of a mathematical problem-solving skills test and a semi-structured interview guide. The research data collection process began with the development of a mathematical problem-solving skills test instrument composed of two descriptive questions. The questions were arranged based on their level of completion, namely easy and medium. A validator for language and material then validated the prepared questions. The validation results from the validator indicated that the test questions were suitable for testing students' mathematical problem-solving skills. Thus, the development stage of the test instrument was completed.

Next, a semi-structured interview guide was developed, tailored to indicators of mathematical problem-solving skills. Therefore, follow-up questions were open-ended and tailored to students' responses during the interview process.

The mathematical problem-solving skills test was administered to students on January 30, 2025. Students were given 60 minutes to complete the given problems. Students were not permitted to use calculators. During the problem-solving process, students were given the freedom to solve the issues and write down their solutions clearly. If the time allotted had expired, students were not permitted to complete the problems. However, if the time had not expired and students had finished and were confident in their answers, they were welcome to collect their answer sheets.

The following day, January 31, 2025, interviews were held for the selected students. Those selected for interviews were the students who provided unique answers among the others. Furthermore, to determine the results of the students' mathematical problem-solving skills test, a mathematical problem-solving skills assessment rubric was developed, as presented in Table 1.

Table 1 The stages of problem-solving questions

Completion Stages	Score	Description
Understanding the Problem	0	Not writing down what is known and asked in the question
	1	Write down what is known and asked in

		the question, but only write part of it on the answer sheet, or it will be incomplete.
	2	Write down what is known and asked in the question in complete and accurate writing.
Devising a plan	0	There is no written settlement plan
	1	Making a resolution plan, but not implementing it, or making a mistake in making the resolution plan
	2	Making a resolution plan can be implemented, but it is not complete, and the results obtained are not accurate, or there are no results.
	3	Making a completion plan can be implemented completely, but the results obtained are not yet accurate.
	4	Create a resolution plan that is implementable, complete, and achieves the right results.
Carrying out a plan	0	Not solving the problem
	1	Inaccurate in solving problems
	2	Correct in solving the problem, but the steps in solving it are not precise, and the results obtained are not accurate.
	3	Correct in solving the problem, the solution steps are proper, but the results obtained are not quite right.
	4	Accurate in solving problems, the solution steps are precise and systematic, and the results obtained are precise.
Looking back	0	There is no inspection done
	1	There are inspections, but not all of them
	2	Carry out a complete inspection

After assessing the students' test scores, the students' answer sheets were scored. Based on the scoring results, six students were selected for interviews. The interviews lasted approximately 30 minutes, depending on the students' answers. The interviews were then recoded based on Polya's theoretical indicators to determine whether the collected data were sufficient.

To ensure the validity and reliability of the research, verification based on data triangulation is necessary. According to Sugiyono (2014) Data triangulation is derived from various sources, including interviews, observations, and the analysis of existing documents. Therefore, in this study, data triangulation was conducted using students' answer sheets and interview transcripts.

Although six students participated in interviews, this article presents the profiles of three participants (HI, SE, and MY). These three were selected because their solution patterns represented the three dominant categories that emerged from the data: (1) reliance on logical–intuitive reasoning (HI), (2) fully structured algebraic modeling (SE), and (3) equation-based reasoning supported by prior learning experience (MY). These profiles captured the widest variation in problem-solving approaches among the six interviewees and were therefore considered the most illustrative for addressing the research objective

RESEARCH RESULTS AND DISCUSSION

To assess the mathematical problem-solving skills of pre-service mathematics teachers, a test instrument consisting of two problems was developed and validated for content and language accuracy. After being confirmed suitable by the validator, the instrument was administered to 20 students on January 30, 2025. The research team then promptly scored and coded the students' answer sheets in preparation for follow-up interviews the next day.

On January 31, 2025, in-depth interviews were conducted to explore students' reasoning processes and the challenges they encountered while completing the test. Six students were selected because their responses showed

distinctive characteristics compared to their peers. Each interview lasted approximately 30 minutes and required students to read aloud the problems and compare their understanding during the interview and with their written solutions. Interview questions were open-ended and aligned with Polya's four problem-solving stages, allowing the researchers to probe students' cognitive processes in detail.

The consolidated data from the test and interviews were then analyzed using the scoring rubric presented in Table 1. This analysis provided a detailed overview of students' performance at each stage of Polya's problem-solving framework. Based on the results of the data analysis test, which assessed the students' mathematical problem-solving skills, the following findings are presented in Table 2.

Table 2. The average mathematical problem-solving skills of students for each question

Question Number	The average problem-solving skills of students
1	84
2	77

Table 2 presents the average scores of students' mathematical problem-solving skills for each test item. The results show clear variation in student performance. Question 1 achieved an average score of 84, indicating a relatively high level of proficiency, whereas Question 2 reached an average of 77, suggesting that students encountered more difficulty with this item. These differences imply that the complexity of each problem influenced how students understood and approached the tasks.

To gain deeper insight into how these performance emerged, the following section analyzes the problem-solving processes of selected pre-service mathematics teachers using Polya's four stages, which include understanding the problem, devising the plan, carrying out the plan, and looking back the results, as the analytical framework. Three participants were chosen because their responses represent distinct and illustrative profiles that reflect the range of strategies, reasoning, and proficiency observed across the dataset. The analysis of these participants (HI, SE, and MY) provides a more detailed portrayal of how students navigate each stage of Polya's framework and helps explain the observed differences in performance across

the test items.

Participant HI

HI is a 20-year-old male and a sixth-semester student in mathematics education. Figure 1 shows the HI answer sheet for solving problem 1.

Phase in understanding the problem, HI demonstrated a basic understanding of the problem by identifying the known and unknown information. However, he did not translate the contextual information into algebraic form. His interpretation relied heavily on intuitive reasoning, indicating a limited connection between the verbal problem structure and formal mathematical representation.

Then, devising a plan, instead of developing a structured mathematical plan, HI relied on logical estimation. His planning was informal, guided by comparing numerical relationships rather than constructing equations. This suggests an intuitive, experience-based approach rather than a strategy grounded in algebraic reasoning.

In phase, while carrying out a plan, HI applied his intuitive logic to generate numerical results, but the steps were not systematic. His written work lacked clarity and organization, making the reasoning process

difficult to follow. This reflects a procedural weakness, particularly when tasks require sequential, symbolic manipulation.

Last, looking back, HI conducted only minimal checking, relying on perceived

“reasonableness” rather than formal verification. He expressed uncertainty about the correctness of his solution because he avoided algebraic formulation. This suggests limited awareness during the evaluation stage.

1. Dik: ... : Dua tahun yang lalu Toni memiliki enam kali dari umurnya bersama Robi.
Delapan belas tahun yang akan datang umur Toni menjadi dua kali umur Robi

Dit: Berapakah umur Toni dan Robi Sekarang?

Jawab: umur Toni dua tahun lalu = $30 : 6 = 5$
umur Robi dua tahun lalu = 5
18 tahun yang akan datang = $18 + 5 = 23$
 $5 \times 2 = 10 \rightarrow$ dua kali umur Robi

Umur Toni Sekarang = 32
Umur Robi Sekarang = 7

Figure 1 Answer Sheet

The following is a transcript of an interview about the completion process written by the student.

Researcher : What do you understand from the first problem?

Student : (reading the problem aloud). If that's (the problem being read), I understand, ma'am. Toni's age is 6 times his son's age.

Researcher: Do you understand the questions asked?

Student : I understand, ma'am. You were asked about Toni and Robi's current age, so if I were to answer it, I would use logic, ma'am, because if I relate it to the formula, I find it difficult, ma'am.

Researcher : Why is it difficult?

Student : I don't understand if you use a formula, ma'am, so it's easier to just use logic. Initially, Toni's age was 6 times his son's age. This means the child's age was 5 years old. After using the 5 years, Robi's age became 30 years old, eh 32 years old, because he added 18 years in the future so his age was 50

years old. After that, his son was 5 years old, twice his age in Robi, which means 5 years multiplied by 5 is 25, then 25 multiplied by 2 is 50. This means the total is the same as 50 years. This means that two years ago the father's age was 32 years old and his son was 7 years old. If you put it in a formula, I don't understand, ma'am. Which formula does this belong to?

Researcher : Was the solution checked again?

Student : There is, ma'am, but I'm not sure.

Researcher : Because I don't use a formula, ma'am.

The interview with HI indicates that he understood the general context of the problem, particularly the age relationship between Toni and his son. However, his explanation shows that this understanding was narrative and intuitive rather than representational or symbolic. When asked to elaborate on his solution process, HI explicitly stated that he avoided using formulas or algebraic expressions because he found them difficult. This suggests that he was unable to

translate verbal information into mathematical form, a key component of Polya's understanding of the problem stage.

During the planning stage, HI relied on informal logical reasoning, using numerical estimates to approximate the father's age, such as the son's age relationship, rather than constructing a mathematical strategy. Although his approach reflects some level of reasoning, it does not align with a structured mathematical plan that Polya emphasizes.

During the carrying out stage of the plan, HI employed a trial-and-error approach based on intuition rather than systematic mathematical operations. The steps he described were inconsistent and lacked a coherent sequence, mirroring the unsystematic nature of his written work.

Regarding rechecking the solution, HI mentioned that he checked his work but remained unsure of its correctness. His uncertainty stemmed from not using a formal mathematical method that could be verified systematically. This reveals that his reflective and evaluative skills, which are central to Polya's fourth stage, were not fully developed.

Overall, the interview provides strong justification that HI relies heavily on informal logical thinking but struggles to transform contextual problems into mathematical models. These findings are consistent with learners at an early stage of mathematical problem-solving development, where intuitive reasoning dominates over symbolic representation.

Based on the answer sheets written by students and interviews conducted, it was found that students tend to have difficulty solving problems using formulas compared to logical thinking. Furthermore, from the interview statements, students stated that making reasonable calculations is a fast way because it allows for comparing two conditions with definite values. The two conditions that were compared gave the same result, so the students concluded that the

calculation based on logic was correct and had been checked repeatedly. The ability to think rationally is often referred to as logical thinking (Faradina & Mukhlis, 2020). The ability of students to think logically can be associated with their thought patterns by comparing two conditions: drawing logical conclusions and answering questions in a logical manner. Thus, students' mathematical problem-solving skills involve logical thinking abilities, where an individual is able to distinguish and criticize events being experienced, whether the event is reasonable and in accordance with science or vice versa, based on certain rules, patterns, and logic possessed by the student (Faradina & Mukhlis, 2020; Usdiyana et al., 2009).

Participant SE

Second, SE is a 20-year-old female sixth-semester pre-service mathematics student. SE provided a distinctive and well-structured solution to Problem 1. In the understanding the problem stage, she clearly identified all known and unknown information. She correctly defined the variables, assigning Toni's age as T and Robi's age as R , and recognized the two temporal conditions given in the problem, two years ago and eighteen years in the future.

During the devising a plan stage, SE translated these conditions into a pair of linear equations:

$$T - 2 = 6(R - 2) \quad (\text{equation 1})$$

$$T + 18 = 2(R + 18) \quad (\text{equation 2})$$

In carrying out the plan stage, she applied the substitution method to solve the system of equations, as shown in Figure 2. Her computation was accurate and logically sequenced.

Finally, in the looking back the results stage, she concluded that Toni's current age is 32 years and Robi's current age is 7 years, although she reported that she did not perform an additional verification because she felt confident in her calculations.

Dik: • 2 tahun yg lalu umur Toni = 6 kali umur Robi.
 • 18 tahun yg akan datang umur Toni = 2 kali umur Robi.

Dit: • berapa umur Toni dan Robi sekarang?

Jawab:

1) Misalkan: • Umur Toni = T
 • Umur Robi = R

2) • dari kondisi 2 tahun yg akan datang

$$= T - 2 = 6(R - 2) \quad \rightarrow \text{Pers I}$$
 • dari kondisi 18 tahun yg akan datang

$$= T + 18 = 2(R + 18) \quad \rightarrow \text{Pers II}$$

3) Penyelesaian Persamaan

Pers I $\rightarrow T - 2 = 6R - 12$

$$T = 6R - 10$$

Pers II $\rightarrow T + 18 = 2R + 36$

$$T = 2R + 18$$

maka substitusi Pers I ke Pers II

$$T = 2R + 18$$

$$6R - 10 = 2R + 18$$

$$6R - 2R = 18 + 10$$

$$4R = 28$$

$$R = 7$$

$$\rightarrow \text{umur Robi 7 tahun}$$

Hitung T

$$T = 6R - 10$$

$$T = 6(7) - 10$$

$$T = 42 - 10$$

$$T = 32 \rightarrow \text{maka umur Toni sekarang 32}$$

∴ Umur Toni saat ini : 32 tahun
 • umur Robi saat ini : 7 tahun.

Figure 2SE Answer Sheet

The following is a transcript of an interview about the completion process written by the student.

Researcher : What do you understand from the first problem?

Student : (reading the problem aloud). From the problem, it is known that Toni's age is 6 times his son's age for the conditions two years ago. Then, 18 years from now, Toni's age will be 2 times his son's age. That's it, ma'am.

Researcher : Do you understand what is being asked?

Student : Understood, ma'am, how old are Toni and Robi now?

Researcher : Can we solve this problem?

Student : Yes, ma'am, by using an example. For example, we symbolize Toni's age with T and Robi's age with R. Then, from what we know there are two conditions, namely the

condition two years ago and the condition eighteen years ago. From these two conditions we can make an equation. Well, for the condition two years ago, that is the first equation (showing equation 1 on the answer sheet). For the condition eighteen years ago, that is the second equation (showing equation 2 on the answer sheet). From these two equations, I find the value of T and R by substitution. Like this, ma'am (the student also reads the substitution process as on the answer sheet in Figure 2).

Researcher : Has the solution been re-examined?

Student : No, ma'am, I'm sure.

The interview with SE provides strong evidence of her ability to apply Polya's

problem-solving stages in a structured and coherent manner. In the understanding of the problem stage, SE demonstrated clear comprehension by accurately restating the two temporal conditions and the age relationships involved. Her explanation shows that she not only recognized the relevant information but also understood how each condition contributed to the structure of the problem.

During the planning a plan stage, SE articulated a formal mathematical strategy. She explicitly defined the variables, T for Toni's age and R for Robi's age, and converted the verbal statements into two linear equations. Her verbal explanation during the interview matched the equations written on her answer sheet, indicating consistency between her conceptual interpretation and symbolic representation.

In carrying out the plan stage, SE described how she used the substitution method to solve the system of equations. She correctly outlined each step, demonstrating strong procedural fluency and confidence in her algebraic manipulations. Her ability to explain the substitution process verbally reflects a well-developed understanding of how the equations relate.

However, in the looking back results stage, SE admitted that she did not check her final answer because she felt certain of its correctness. While such confidence aligns with her accurate and well-organized reasoning, it also indicates a partial application of Polya's framework, as systematic verification is a critical final step in the problem-solving process.

Overall, the interview data justify categorizing SE as a structured symbolic solver who is a participant who demonstrates strong conceptual understanding and procedural skills, but tends to overlook the importance of reflective checking in the final stage of Polya's method.

The evidence from Figure 2 and the interview with SE consistently demonstrates that she applied Polya's four problem-solving stages in a structured and coherent manner, which aligns with Polya's (1985) emphasis on systematic progression through each step. Her written work, as shown in Figure 2, demonstrates a clear and accurate conversion of the problem's verbal statements into algebraic form. Representing contextual

information using symbols, such as defining Toni's age as T and Robi's age as R, reflects strong symbolic reasoning, a competency highlighted in prior research as essential for successful mathematical modeling (Foster, 2023; Hiebert et al., 1996; Jupri et al., 2024). This alignment between context and symbolic representation indicates a solid understanding of the problem stage.

SE's interview responses further reinforce the correctness of her planning stage. She explicitly explained how she interpreted each condition and justified the formulation of the two equations shown in Figure 2. The coherence between her verbal explanation and written solution demonstrates both procedural fluency and conceptual clarity, indicating that her abilities support accurate problem-solving in pre-service teachers (Christina & Adirakasiwi, 2021; Fatmala et al., 2020).

The carrying out the plan stage is also well supported by Figure 2. SE solved the system of equations using the substitution method, performing each computation correctly and presenting the steps in a logical manner. Her confident explanation during the interview reflects the type of algebraic fluency deemed critical in mathematical problem solving (Widjajanti, 2009). The consistency between her written and verbal responses suggests that she understood not only how to execute the procedure but also why each step was necessary, in line with recommendations for effective problem-solving instruction (Hartono, 2020).

However, both the written work and interview reveal a limitation in the looking back the results stage. Although SE arrived at the correct final answer, she acknowledged that she did not perform a formal re-check because she felt confident in her calculations. This pattern mirrors previous findings that students often neglect the verification step of Polya's framework (Kania & Fitriyani, 2022; Putri et al., 2022; Yayuk & Husamah, 2020). This suggests that, despite her strong conceptual and procedural skills, her reflective evaluation remains underdeveloped.

Overall, the convergence between Figure 2 and the interview data justifies categorizing SE as a high-performing symbolic problem solver who excels in translating, modeling, and executing mathematical strategies, yet shows limited

engagement in systematic verification (Sumarni et al., 2021; Widjajanti, 2009).

Participant MY

Third, MY is a 20-year-old female pre-service mathematics teacher in her sixth semester. MY provided a distinctive solution to Problem 1. Although she did not explicitly write the known and unknown information, she demonstrated an understanding of the problem by immediately transforming the verbal statements into symbolic form, assigning the father's age as x and the child's age as y , as shown in Figure 3.

In the understanding the problem stage, MY identified the two conditions embedded in the problem. In the first condition, such as two years ago, she formulated the equation $x - 2 = 6(y - 2)$ and correctly performed the associated algebraic steps. In the second condition—eighteen years in the future, she wrote the equation $x + 18 = 2(y + 18)$ and again executed the algebraic operations accurately, resulting in two well-formed linear equations.

During the planning and carrying out the plan stages, MY chose the elimination strategy to solve the system of equations. She applied this method correctly, obtaining $y = 7$ and $x = 32$. Her steps were systematic and reflected a clear understanding of algebraic manipulation.

In the final stage, looking back, MY concluded that Toni's current age is 32 years and Robi's current age is 7 years. Although her conclusion was correct, it remains unclear whether she rechecked her solution, as no verification steps were written on the answer sheet.

Overall, MY's work demonstrates strong conceptual understanding and procedural competence. Her ability to translate the problem into symbolic form, construct correct equations, and implement an appropriate strategy indicates effective engagement with the first three stages of Polya's framework, despite the absence of explicit verification in the final stage.

1. Diketahui: Umur ayah: x
Umur anak: y
ditanya: Umur Toni dan Robi?

• 2 tahun yang lalu • 18 tahun mendatang

$$x - 2 = 6(y - 2)$$

$$x - 2 = 6y - 12$$

$$x - 6y = -12 + 2$$

$$x - 6y = -10 \dots (i)$$

$$x + 18 = 2(y + 18)$$

$$x + 18 = 2y + 36$$

$$x - 2y = 36 - 18$$

$$x - 2y = 18 \dots (ii)$$

• substitusikan pers (i) & (ii)

$$x - 2y = 18 \dots (ii)$$

$$x - 6y = -10 \dots (i)$$

$$\underline{x - 2y = 18}$$

$$x - 6y = -10$$

$$4y = 28$$

$$y = \frac{28}{4}$$

$$y = 7$$

$$x - 6y = -10$$

$$x - 6(7) = -10$$

$$x - 42 = -10$$

$$x = -10 + 42$$

$$x = 32$$

Jadi, ~~umur~~ Umur Toni adalah 32 tahun dan Robi berumur 7 tahun

Figure 3 MY Answer Sheet

To examine MY's problem-solving process in more depth, a follow-up interview was conducted.

Researcher : What do you understand from the first problem?

Student : (reading the problem aloud).
From what I understand from the problem, the important thing is to analyze how old Toni is using a system of

linear equations in two variables, so the age of Toni's son, Robi, is six times Toni's age. If using system of linear equations in two variables, Toni's x is minus two because 2 years ago, for Robi, 6 times y is minus 2.

Researcher : Do you understand what is being asked?

Student : Understood, ma'am. Toni and

Robi's ages.

Researcher : How did you solve it?

Student : Because we already have the forms of Toni's age x and Robi's age y , we immediately thought of system of linear equations in two variables, ma'am. We know two equations, ma'am. Then we eliminate the equations, ma'am. Then we get the age $y = 7$. Then we do the calculation again, ma'am, and we get $x = 32$.

Researcher : What was done in the completion process? Was there any checking of the answers?

Student : Just checked the second equation, ma'am. Because I wasn't sure yet, ma'am.

Researcher : How to check the overall answers?

Student : No, ma'am.

Researcher : Have you ever worked on a problem like this?

Student : Yes, ma'am, when I was in junior high school, the teacher once gave me this question.

When asked about her understanding of the first problem, MY immediately associated it with a system of linear equations in two variables. She explained that she interpreted the conditions by assigning Toni's age as x and Robi's age as y , and then constructed the expressions about $x - 2$ and $y - 2$ to represent the situation two years ago. This response indicates that MY understood both the structure and intent of the question. She also correctly identified that the problem asked for the current ages of Toni and Robi.

In describing her solution method, MY stated that once the variables were determined, she proceeded to form two linear equations corresponding to the two conditions. She then applied the elimination method to solve for the unknowns, first obtaining $y = 7$, and subsequently calculating $x = 32$. This demonstrates that MY had a clear and systematic plan for solving the problem, consistent with the planning and carrying out stages of Polya's

framework.

Regarding the looking back at the results stage, MY admitted that she only checked the second equation because she felt uncertain about it, and did not verify the entire solution. This partial verification suggests that while she understood the procedural steps, her reflective evaluation remained incomplete. The interview confirms that MY's thought process aligned closely with the written solution on her answer sheet, reflecting the ability to execute a coherent sequence of algebraic operations.

The overall interview findings indicate that MY possesses strong problem-solving skills. She demonstrated accurate problem interpretation, formulated appropriate mathematical models, selected an effective solution strategy, and carried out the computations correctly. Although her verification process was limited, her explanation reflects a solid understanding of algebraic concepts, which supports further development of her problem-solving skills, consistent with the findings of Rizqiani et al., (2023), and Shiddiq & Herman, (2023). Furthermore, MY mentioned that she had encountered a similar problem during junior high school, enabling her to recall and transfer prior knowledge when solving the current task. This aligns with (Steiner, 2001), who emphasizes that learners' ability to draw on previously acquired knowledge plays a critical role in successful problem solving and in the broader process of knowledge transfer.

CONCLUSION

The findings of this study indicate that pre-service mathematics teachers exhibit diverse profiles of mathematical problem-solving skills, influenced primarily by differences in prior experience and conceptual understanding. Prior experience can support effective problem representation, planning, and execution across Polya's stages; however, it does not necessarily ensure systematic engagement in the looking back stage. This highlights the need for structured instructional approaches that explicitly foster reflective verification in problem solving. By identifying distinct problem-solving profiles, mathematics teacher education programs can provide targeted support to strengthen both procedural competence in mathematical problem-solving skills, thereby better preparing future mathematics teachers to develop problem-solving skills in their own

classrooms.

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