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STUDENTS' CRITICAL THINKING AT INFORMAL DEDUCTION LEVEL IN PISA-LIKE PROBLEMS ON SPACE AND SHAPE CONTENT

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Abstrak

Berpikir kritis merupakan kemampuan seseorang dalam mengenali, menganalisis, dan menyelesaikan masalah. Indikator berpikir kritis yaitu *focus, reason, inference, situation, clarity*, dan *overview*. Penelitian ini dilakukan untuk mengetahui kemampuan berpikir kritis siswa dengan level berpikir Deduksi Informal dalam menyelesaikan soal model PISA konten *space and shape* materi Lingkaran. Penelitian berjenis deskriptif kualitatif dengan subyek siswa kelas XI MA di Jember. Instrumen yang digunakan yaitu *Van Hiele Geometry Test* (VHGT), soal model PISA, dan pedoman wawancara. Analisis data dilakukan dengan langkah reduksi data, penyajian data, dan penarikan kesimpulan. Keabsahan data dilakukan melalui triangulasi teknik dan sumber. Pada tahap awal, dilakukan VHGT pada 26 siswa untuk menentukan level berpikirnya. Kemudian dipilih 2 siswa pada Level Deduksi Informal dengan kriteria kemampuan matematika tinggi dan komunikasi yang baik. Berikutnya dilakukan tes untuk mengetahui kemampuan berpikir kritisnya. Hasil analisis data menunjukkan bahwa siswa dengan level berpikir deduksi informal mampu memenuhi kriteria berpikir kritis *focus, reason, inference, situation, clarity, dan overview*.
Kata kunci: Berpikir Kritis, Level Berpikir Van Hiele, Soal Model PISA

Abstract

Critical thinking is a person's ability to recognize, analyze, and solve problems. Indicators of critical thinking are focus, reason, inference, situation, clarity, and overview. This study was conducted to determine the critical thinking ability of students with the Informal Deduction thinking level in solving PISA-like problems on the content of space and shape on the material of Circles. This research is a qualitative descriptive study with subjects of grade XI students of Madrasah Aliyah at Jember. The instruments used were the Van Hiele Geometry Test (VHGT), PISA-like problems, and interview guidelines. Data analysis included data reduction, data presentation, and conclusion. Data validity was carried out through triangulation of techniques and sources. In the initial stage, VHGT was carried out on 26 students to determine their level of thinking. Then 2 students were selected at the Informal Deduction Level who have high mathematical ability and good communication. Next, a test was carried out to determine their critical thinking abilities. The results of the data analysis showed that students with the informal deduction thinking level were able to focus, reason, inference, situation, clarity, and overview.
***Keywords**: Critical thinking, Van Hiele Thinking Level, PISA-like problem*

INTRODUCTION

Education is interpreted as a process that includes interactions between teachers and students to achieve certain goals (Aini et al., 2020). Therefore, education is an important factor in building a person's character in society. Education is also interpreted as a human effort to develop their abilities, both in science and technology. The rapid advancement of technology cannot be separated from the collaboration between mathematics and sciences (Annizar et al., 2020). Thus, mathematics plays an important role in real life.

Students who study mathematics are expected to have the ability to answer various problems encountered in everyday life (Aini et al., 2020). One of the abilities that students are expected to master is critical thinking (Putri et al., 2023). Critical thinking skills are a mental process for making decisions correctly that requires the skills to recognize

and examine the problems encountered, evaluate and apply previously owned information, and recognize the interrelationships of various things (Putri et al., 2023). Critical thinking includes high-level thinking skills that include various processes in processing information in the context of decision-making or situations or problems encountered by a person (Noviastuti et al., 2024).

The criteria of critical thinking are focus, reason, inference, situation, clarity, and overview (Ennis, 1962). Focus is the ability to focus the question in the problem to make decisions about what to believe (Ennis, 1962, Ayu et al., 2022). Reason is the ability to found the reason that support or reject decisions made based on the facts in the problem. Inference is the ability to make reasonable or convincing conclusions (Ennis, 1962, Ayu et al., 2022). Situation is the ability to understand the situation and keeping the situation in mind to help clarify questions and understand the meaning of supporting decisions taken(Ennis, 1962, Ayu et al., 2022). Clarity is the ability to explain the meaning or terms used. Overview is the ability to review and throughly researching the decisions take (Ennis, 1962, Ayu et al., 2022).

Critical thinking ability can be trained by solving math problems (Putri et al., 2023). One of the questions can be used is PISA-like problems (Septiadi et al., 2020). It's a question that requires a lot of analysis from students who are working on it (Septiadi et al., 2020).

The four-yearly assessment namely Program for International Student Assessment (PISA) shows low students' abilities in mathematics. The results of PISA 2022 placed Indonesia in 66th place out of 81 participating countries (https://www.oecd-ilibrary.org/). One of the causes is that the PISA-like problems are unfamiliar to students. Therefore, this study will use PISA-like problems to introduce this type of question to students. The questions tested on PISA cover four contents: change and relationship, space and shape, quantity, and uncertainty and data (https://pisa2022-maths.oecd.org/). Mostly, PISA-like problems represented in a daily life context (Septiadi et al., 2020). Due to limitations in this study, the PISA-like problems used were only on the space and shape content. This content is related to distance, shape, and object visualization. Thus, the material related to this content is geometry.

Geometry is considered as a part of mathematics that has an important role because it can help someone interpret and understand the surrounding environment (Aini et al., 2020). Geometry can improve logical thinking and making generalizations correctly (Pujiastuti & Haryadi, 2024). Therefore, geometry is an important topic in school mathematics.

Previous studies as conducted by Pujiastuti & Haryadi (2024) reveal that geometry is one of the subjects that is hard for students to understand. Other previous research by Firmanti et al (2024) state that the geometry ability of high school students is also still low and students can only understand the material according to their thinking level as explain in the Van Hiele theory (Pradana & Nalim, 2024).

Students' abilities in geometry shown by the Van Hiele Thinking Level. Van Hiele stated that studying geometry involves five levels: level 0 (Visualization), level 1 (Analysis), level 2 (Informal Deduction), level 3 (Formal Deduction), and level 4 (Rigor).

The visualiazation level described as the ability to identify geometric shapes, draw and immitate drawing (Tao & Fu, 2024, Firmanti et al., 2024). The analysis level marked by the ability to identify and generalize the propertise of spesific geometric shapes to solve problems through informal analysis of various shapes (Tao & Fu, 2024, Firmanti et al., 2024). Informal deduction is a stage that students can understand geometric sequences or the relationship of various shapes (Tao & Fu, 2024, Firmanti et al., 2024). They can correlate shapes with therir properties, formulate informal deduction, and understand the elements that constitute shapes (Tao & Fu, 2024, Firmanti et al., 2024). Formal deduction is a level that students can appreciate the significance mathematical proof and demostrate solutions to geometris problems through abstract reasoning (Tao & Fu, 2024, Firmanti et al.,

2024). Rigor as the highest level, described as the ability to understand the importance of accuracy from the most basic things and use theory and postulates in understanding geometric concepts (Tao & Fu, 2024, Firmanti et al., 2024).

The thinking level of high school students in learning geometry according to the Van Hiele level has reached level 2 (Informal Deduction) (Wulandari & Ishartono, 2022, Zurriatinnisa, 2024). Many students do not learn the basic concepts of geometry at the junior high school, resulting in their failure to reach levels 3 and 4 (Usiskin, 1982). This means that in general, the thinking level of high school students is only able to reach a maximum of level 2. Based on the previous explanation, this study will describe students' critical thinking skills at the Informal Deduction level in solving PISA model problems.

METHODS

This study uses a descriptive qualitative approach. The subjects of the study were students of class XI Madrasah Aliyah at Jember who were selected by purposive sampling. In the initial stage, VHGT was given to assess the students' thinking level. The VHGT used refers to the instrument developed by The Cognitive Development and Achievement in Secondary School Geometry Project (CDASSG) (Usiskin, 1982). Then 2 students were selected at the Informal Deduction Level who have high mathematical ability and good communication to take the critical thinking ability test. Two questions used in this test used PISA-like problems on space and shape content on Circle material. The critical thinking ability test instrument used has validity and reliability tested. The validity coefficient obtained was 3.9 and the reliability coefficient was 0.649 so that it was valid and reliable. After the test, the two subjects were interviewed based on the interview guidelines that had been prepared.

The questions used in this study are presented in Figure 1. Data analysis used the Miles and Huberman model included data reduction, data presentation, and conclusions. Data reduction is done by sorting, simplifying, and selecting relevant data from the results of observations, interviews, and documentation. Data that is not relevant to the research needs is removed to ensure data clarity. Furthermore, data presentation is done in the form of narratives and interview transcripts. Conclusions are drawn based on the analysis of data results.

1. Sebuah kapal induk sedang berlayar ditengah lautan yang luas. Kapal induk ini terletak pada koordinat (3, 4) dan dilengkapi dengan teknologi radar yang mampu mendeteksi kapal lain hingga jarak 20 km ke segala arah. Dengan memiliki radar berdaya jangkauan luas, kapal-kapal ini dapat memonitor kondisi sekitar dan mengidentifikasi potensi resiko tabrakan. Selain kapal induk, nampak dari kejauan ada lima kapal kecil yang kemungkinan sedang mencari ikan. Kapal-kapal kecil tersebut letaknya saling berjauhan antara kapal satu dengan kapal lainnya. Jika letak koordinat kapal kecil tersebut dapat dilihat pada tabel berikut:

Kapal	Letak Koordinat Kapal Kecil
Kapal A	(14, 20)
Kapal B	(15, -6)
Kapal C	(10, 25)
Kapal D	(12, 22)
Kapal E	(-10, 10)

Dari kelima kapal kecil, manakah kapal kecil yang dapat dideteksi oleh radar kapal induk ? jelaskan alasanmu!

2. Kota yang berada di tepi pantai diwarnai oleh keindahan panorama laut yang menakjubkan. Sehingga banyak wisatawan dari berbagai kota berlibur dikota ini. Para wisatawan luar kota ini tidak hanya menggunakan mobil dan sepeda motor untuk berlibur, tetapi juga menggunakan kereta api untuk menghindari kemacetan. Kota ini memiliki empat jalur kereta api yang berbeda menuju ke berbagai kota disekitarnya. Jalur kereta api ini membentuk suatu persamaan garis. Berikut jalur kereta api yang membentuk persamaan garis:

Jalur kereta api	Persamaan garis
Jalur pertama	$x + y = 3$
Jalur kedua	$y = x$
Jalur ketiga	$x + y = 5$
Jalur keempat	$-x + y = 4$

Namun, ketenangan kota tersebut terganggu oleh gempa bumi yang tiba-tiba terjadi. Gempa bumi tersebut menyebabkan kerusakan pada beberapa wilayah. Daerah yang terdampak oleh gempa membentuk lingkaran dengan persamaan lingkarannya $x^2 + y^2 - 6x + 14y + 9 = 0$. Manakah dari empat jalur kereta api yang terdampak oleh gempa? Jelaskan alasanmu!

Figure 1. PISA-like problem to assess critical thinking

RESULTS AND DISCUSSION

The results of data analysis on VHGT in 26 students are presented in the following Figure 2. It can be seen from Figure 2 that no students reached the Formal Deduction and Rigor levels. This finding is in line with previous research that at the high school level, only subjects were found at the highest level of Informal Deduction (Zurriatinnisa, 2024). As many as 19% of students only reached the pre-visual level, which could be due to not understanding the geometry subject, rushing to complete the test, not being serious about taking the test, and not reading the questions properly (Fitriyani et al., 2018). Some strategies to advance Van Hiele Level from Informal to Formal deduction are rigorization of natural reasoning, symbolization of geometric reasoning, and understanding logical thinking and its constituent elements (Tao & Fu, 2024).

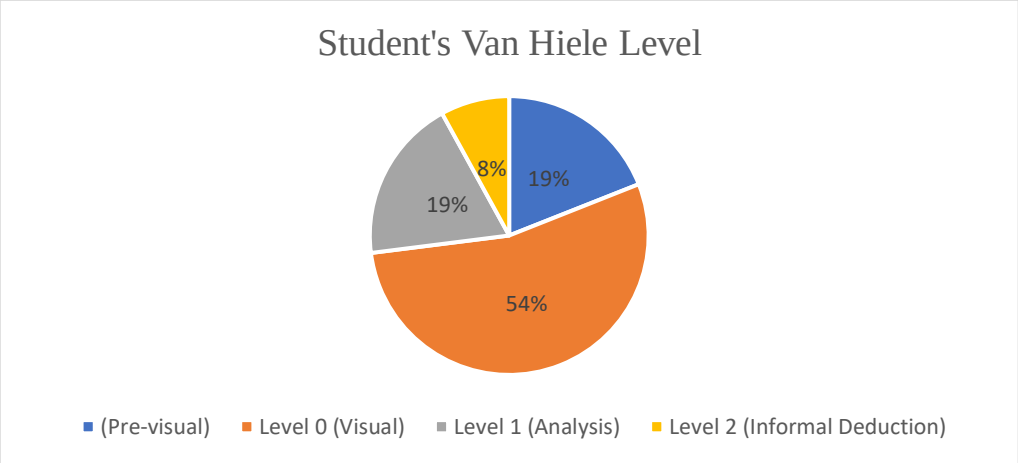

Figure 2. VHGT results

The next stage was to select two students at Informal Deduction level to take the critical thinking ability test. The selection of these two subjects was based on equivalent mathematics abilities, as evidenced by the mathematics scores on the Daily Test, Mid-Term Exams, and Final Exams.

Next, a critical thinking ability test was carried out. The results are described as follows. Subject 1's answer to question number 1 is presented in Figure 3 below.

From Figure 3, it can be seen that Subject 1 wrote down the correct information regarding the coordinates of the aircraft carrier, radar range, and the coordinates of each. So it can be said that she met the focus criteria. Next, she wrote the circle equation formula and substituted the coordinates of the aircraft carrier and radar range, so that she met the reason criteria. Next, she determined the ship detected by the radar. Triangulation with interview data showed that she met the inference criteria. She also met the situation indicators that appeared when she used all the information related to the problem, including information that was not taught. Subject 1 also wrote a statement that the small ships detected by the aircraft carrier were Ship A, Ship B, and Ship E and the argument that Ship C and Ship D were not detected by the aircraft carrier. Thus, she met the Clarity criteria as evidenced by his ability to explain the conclusions he wrote on the answer sheet. For the last criteria, he was proven to be able to re-check the process of working on the questions from start to finish. This is evident from the triangulation of interview data.

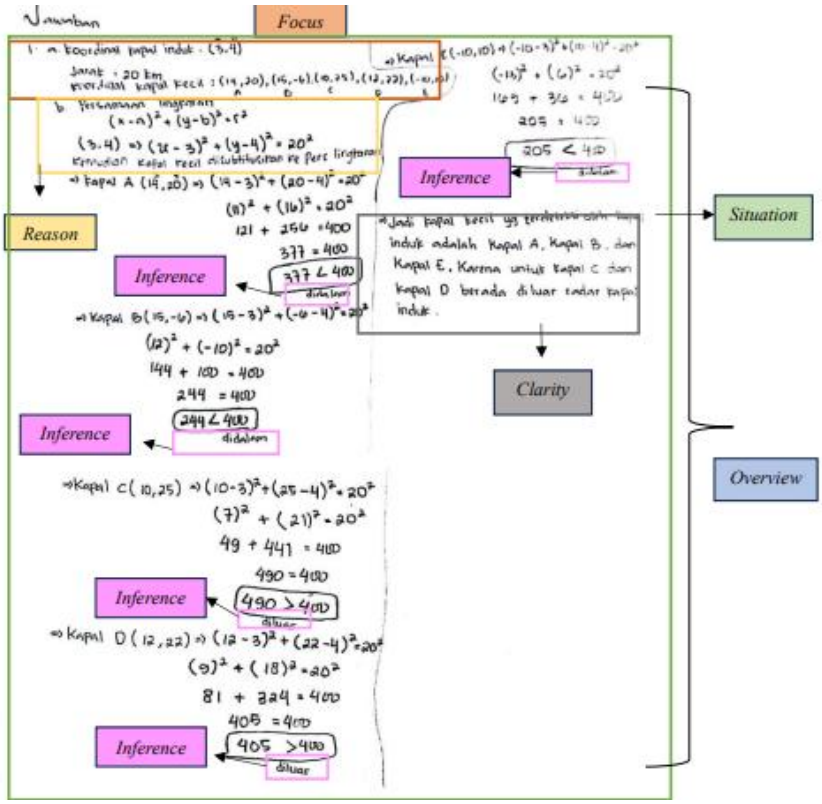

Figure 3. Subject 1's answer to question number 1

Subject 1's answer to question number 2 is presented in Figure 4. Based on Figure 4, it appears that Subject 1 can write down the correct information about the question, namely about the lines of each train and the equation of the circle. It can be said that Subject 1 meets the focus criteria. In the next stage, he wrote a description of how to work on the second question by substituting the train lines one by one into the circle equation. He was also able to provide reasons for each process of solving the question. Thus, he met the reason criteria. He also provided information on which train lines were affected by the earthquake. Triangulation of interview data also showed that he could conclude the answers correctly so that he met the inference criteria. The figure also shows that Subject 1 substituted each train line into the equation of the circle. This strengthens the evidence that he understands the mathematical problem and how to solve it to conclude. It can be said that he meets the situation criteria. Furthermore, Subject 1 provided information about the train lines affected by the earthquake. Therefore, he meets the clarity criteria. Meanwhile, the overview criteria have been met with evidence that he can show the process of re-checking the answers that have been written.

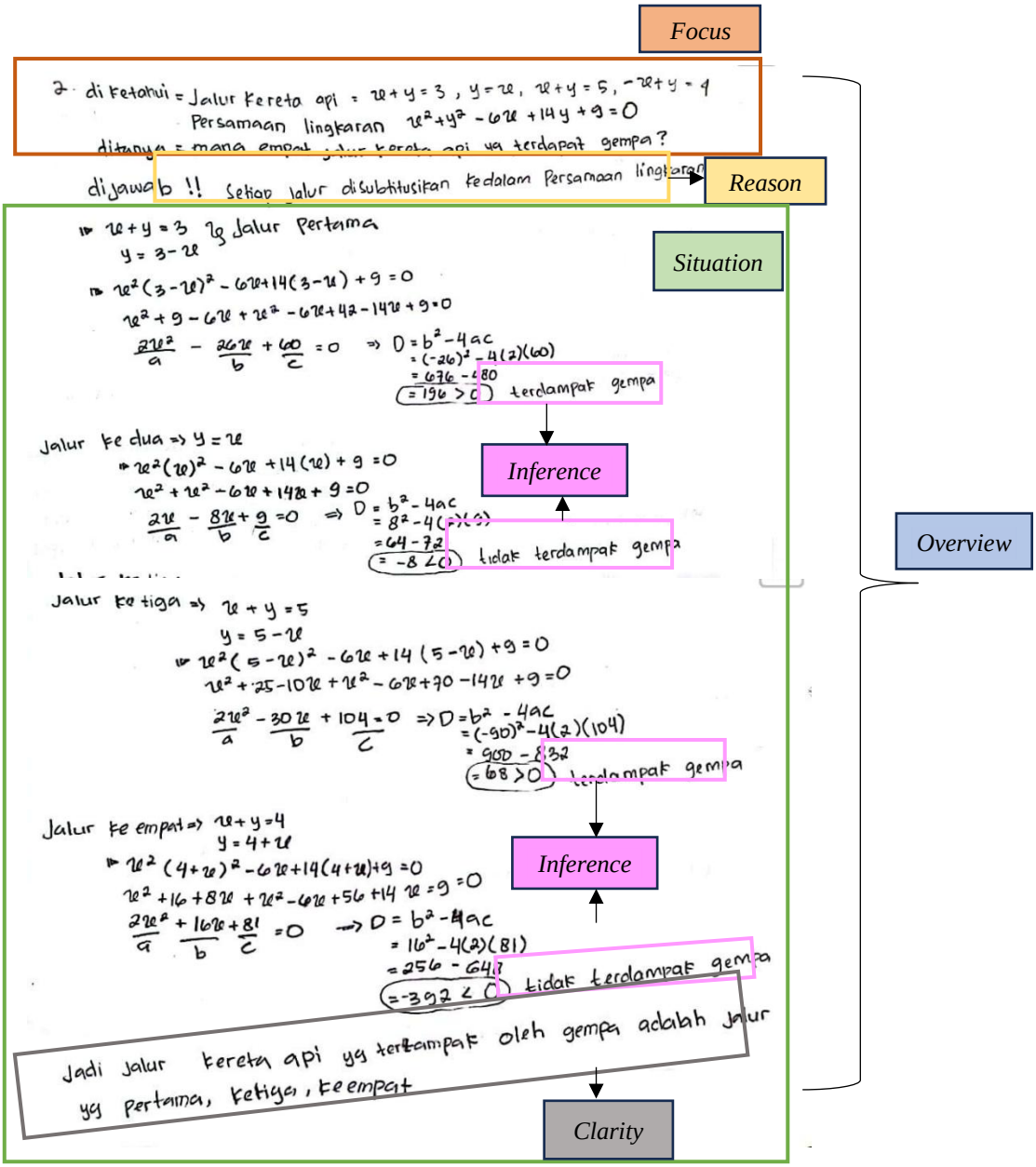

Figure 4. Subject 1's answer to question number 2

The following presents the answer to Subject 2 for question number 1 in Figure 5. Similar to the previous subject, Subject 2 also demonstrated the ability to write question information on the answer sheet. He wrote the coordinates of the aircraft carrier, the radar range, and the coordinates of all small ships. It can be said that he met the focus criteria. The next step was to write a circle equation and substitute the coordinates of the aircraft carrier and the radar range. Triangulation with interview data showed that he met the reason criteria, as evidenced by his ability to provide arguments at each stage of the conclusion. Subject 2 also substituted the coordinates of small ships and determined which small ships might be detected by radar. It is said that he met the inference criteria. The procedure for working on the questions carried out by subject 2 by creating a circle equation using the coordinates of the aircraft carrier and the radar range of up to 20 km to be substituted into the equation showed that he met the situation criteria. Subject 2 was also able to provide information on which ships were detected by radar, so it can be said that he met the clarity criteria, namely the ability to

explain the process of drawing conclusions and terms in the questions. The results of the test and interview also showed that he met the overview criteria, as evidenced by his ability to re-check the answers he wrote.

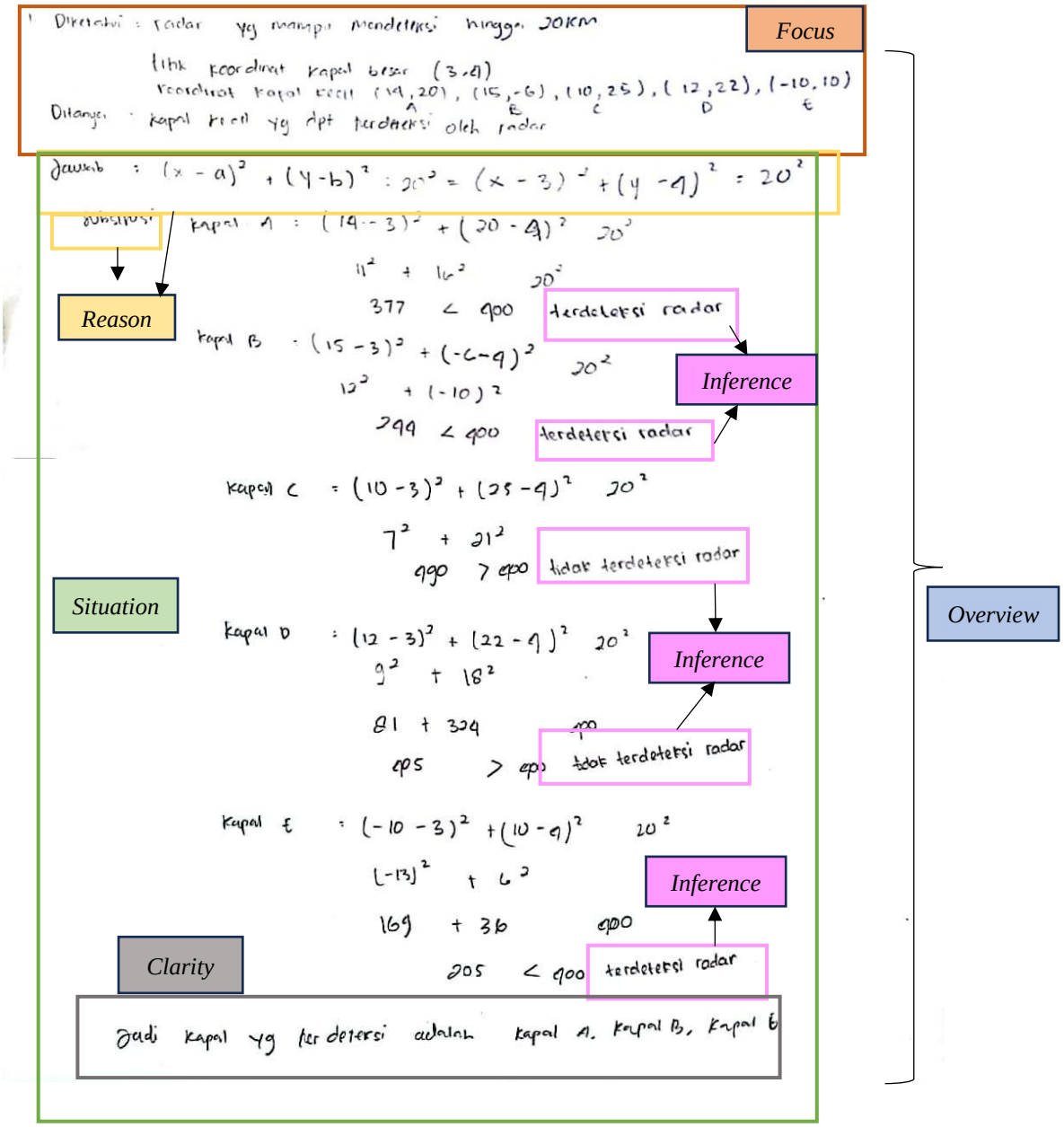

Figure 5. Subject 2's answer to question number 1

Next is Subject 2's answer to question number 2 which is presented in Figure 6.

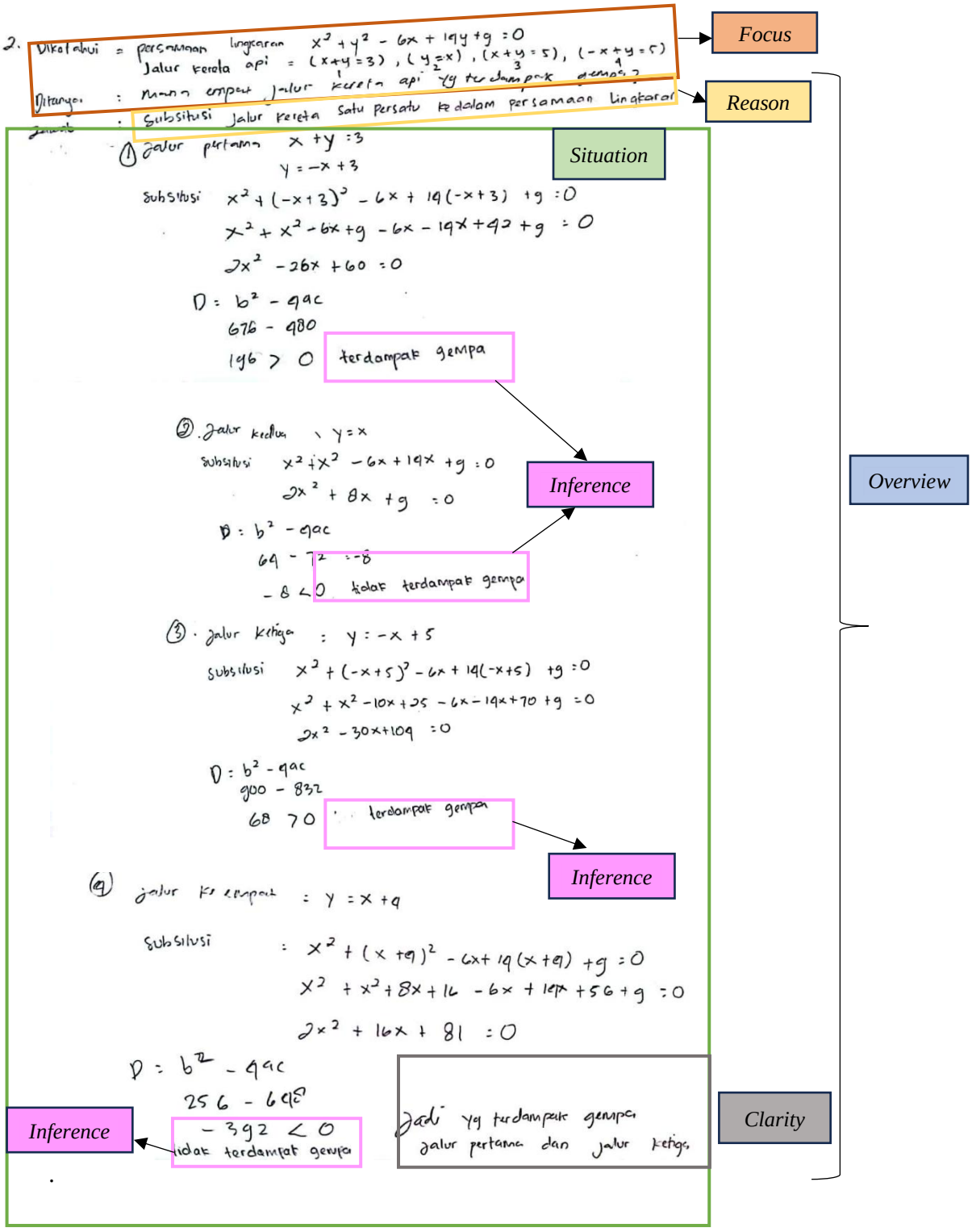

Figure 6. Subject 2's answer to question number 2

Based on the results of the test and interview, it can be seen that the informal deduction subject can meet 6 criteria for critical thinking skills, namely focus, reason,

inference, situation, clarity, and overview. This subject can state what is known and asked correctly and precisely and can write the solution method correctly. The informal deduction subject can provide further explanation of the conclusions made. But have not been able to create similar examples of questions. It was stated by previous research that students at the informal deduction level can understand the problem well because students can understand the question sentence well, and know exactly what is known and what is asked in the question. Students can solve problems according to the correct solution strategy and the correct calculation process (Pebruariska & Fachrudin, 2018). As stated previously, that students at Informal Deduction can explore the intrinsic attributes of specific shapes and the potential relationships between various shapes. They begin to classify shapes and use formulas, definitions, or learned properties to make informal deductive inferences (Tao & Fu, 2024). They also can understand geometric shapes sequences or the relationship between them (Pipit Firmanti et al., 2024). Thus, they could meet all of critical thinking criteria.

CONCLUSION

The results of the data analysis revealed that the highest Van Hiele thinking level only reached Informal Deduction. Students at the Informal Deduction level showed good critical thinking skills. The criteria that are met, included focus, reason, inference, situation, clarity, and overview.

REFERENCES

Aini, A. N., Mukhlis, M., Annizar, A. M., Jakaria, M. H. D., & Septiadi, D. D. (2020). Creative thinking level of visual-spatial students on geometry HOTS problems. *Journal of Physics: Conference Series*, *1465*(1), 012054. https://doi.org/10.1088/1742-6596/1465/1/012054

Amri, A. A. U., & Mustika, J. (2022). FRISCO Criteria: Analysis of Student's Mathematic Critical Thinking Ability on HOTS Test. *International Conference on Language, Linguistics, Literature and Education (ICLLLE)*, 205-219, Lampung: Universitas Teknokrat Indonesia

Annizar, A. M., Masrurotullaily, Jakaria, M. H. D., Mukhlis, M., & Apriyono, F. (2020). Problem solving analysis of rational inequality based on IDEAL model. *Journal of Physics: Conference Series, 1465*(1), 012033. https://doi.org/10.1088/1742-6596/1465/1/012033

Ennis, R. H. (1962.). A Concept of Critical Thinking a Proposed Basis for Research in the Teaching and Evaluation of Critical Thinking Ability. *Harvard Educational Review*, 32(1), 81-111.

Firmanti, P. Yuberta, F., Septiadi, D. D. & Nisa, N. R. (2024). Geometry ability in Senior High School Students: Based on Learning Style. *Hipotenusa: Journal of Mathematical Society*, *6*(1), 88–100. https://doi.org/10.18326/hipotenusa.v6i1.1901

Fitriyani, H., Widodo, S. A., & Hendroanto, A. (2018). Students' Geometric Thinking Based on Van Hiele's Theory. *Infinity Journal*, *7*(1), 55-60. https://doi.org/10.22460/infinity.v7i1.p55-60

Noviastuti, N. D., & Aini, A. N. (2024). Pengaruh Contextual Teaching and Learning Berbasis Budaya Suku Osing Terhadap Kemampuan Berpikir Kritis Siswa. *Journal Of Mathematics Learning Innovation (JMLI)*, *3*(2), 90–100. https://doi.org/10.35905/jmlipare.v3i2.10572

Pradana, G. B. & Nalim, N. (2024). The Effect of Mathematics Learning with the Creative Visualization Technique on the Geometry Ability Level of Students at SMP Islam Wonopringgo. *AB-JME: Al-Bahjah Journal of Mathematics Education*, *2*(2), 100–109. https://doi.org/10.61553/abjme.v2i2.206

Pebruariska, A., & Fachrudin, A. D. (2018). Kemampuan Pemecahan Masalah Siswa Kelas VII pada Materi Segiempat ditinjau dari Tingkat Berpikir Geometri Van Hiele. *AKSIOMA : Jurnal Matematika Dan Pendidikan Matematika*, *9*(1), 21-28. https://doi.org/10.26877/aks.v9i1.2461

Pujiastuti, H., & Haryadi, R. (2024). The Effectiveness of Using Augmented Reality on the Geometry Thinking Ability of Junior High School Students. *Procedia Computer Science*, *234*, 1738–1745. https://doi.org/10.1016/j.procs.2024.03.180

Putri, I. S., & Aini, A. N., (2023). Analisis Kemampuan Berpikir Kritis Siswa dalam Menyelesaikan Soal HOTS Ditinjau Dari Keaktifan Pada Pembelajaran Cool-Critical-Creative-Meaningful. *Math Educa Journal*, 7(1), 1-10.
https://ejournal.uinib.ac.id/jurnal/index.php/matheduca/article/view/5855

Septiadi, D. D., Kholil, M., Masrurotullaily, Apriyono, F., & Aini, A. N. (2020). Design of PISA-liked problem which used jember fashion carnival context to train students' analytical thinking. *Journal of Physics: Conference Series*, *1465*(1), 1-13. https://doi.org/10.1088/1742-6596/1465/1/012072

Tao, S., & Fu, H. (2024). An Investigation into the Challenges and Strategies of Secondary Students' Geometric Thought Processes through the Lens of Van Hiele's Theory. *Reviews*, *6*(6), 228-231. https://doi.org/10.12238/rerr.v6i6.2252

Usiskin, Z. (1982). Van Hiele Levels and Achievement in Secondary School Geometry *CDASSG PROJECT*. Chicago: The University of Chicago. https://files.eric.ed.gov/fulltext/ED220288.pdf

Wulandari, T. A., & Ishartono, N. (2022). Analisis Kemampuan Representasi Matematika Siswa SMA Dalam Menyelesaikan Soal Geometri Berdasarkan Level Berpikir Van Hiele. *JNPM (Jurnal Nasional Pendidikan Matematika)*, *6*(1), 97. https://doi.org/10.33603/jnpm.v6i1.5330

Zurriatinnisa, S. (2024). Kesulitan Siswa dalam Menyelesaikan Soal Setara PISA konten Shape and Space ditinjau Berdasarkan Level Tingkat Berpikir van Hiele. *Syntax Literate; Jurnal Ilmiah Indonesia*, *9*(1), 100–110. https://doi.org/10.36418/syntax-literate.v9i1.14221