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# MATHEMATICAL CERTAINTY IN CONTEMPORARY FORMAL SYSTEMS: A SYSTEMATIC LITERATURE REVIEW OF AXIOMATIC STRUCTURES, PROOF THEORY, LOGICAL ARCHITECTURES, AND COMPUTATIONAL VERIFICATION

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**Abstract**

Mathematical certainty in formal mathematics is commonly associated with the validity, consistency, and correctness of reasoning within explicitly defined formal systems. However, recent developments in axiomatic foundations, proof theory, non-classical logic, and computational verification suggest that certainty is increasingly discussed as a structured and system-relative phenomenon. This study aims to synthesize how mathematical certainty is conceptualized in contemporary literature through axiomatic structures, proof-theoretic mechanisms, logical architectures, and computational verification. A systematic literature review was conducted using the PRISMA framework. Twenty peer-reviewed studies published between 2020 and 2026 and retrieved from the Scopus database were selected based on predefined inclusion and exclusion criteria. Thematic synthesis identified four interrelated dimensions: axiomatic foundations as formal constraints, proof-theoretic mechanisms as procedures for validating derivations, logical architectures as system-relative frameworks of inference, and computational verification as a mechanism for strengthening reproducibility in formal proof validation. The findings suggest that mathematical certainty in the reviewed literature is not treated solely as a fixed metaphysical guarantee, but may be interpreted as a structurally mediated condition supported by coherence, rule-governed derivation, logical validity, and verifiable formalization. This review contributes a conceptual framework for understanding mathematical certainty within contemporary formal mathematical systems while acknowledging the limitations of a Scopus-based corpus.

**Keywords:** Axiomatic Systems, Computational Verification, Formal Logic, Mathematical Certainty, Proof Theory

## INTRODUCTION

Mathematical certainty has traditionally been associated with deductive reasoning grounded in axiomatic systems and formal proof (Giovannini & Schiemer, 2025; Quinn, 2024). In this review, mathematical certainty is understood as the justified reliability of mathematical claims within explicitly defined formal systems. It refers to the extent to which mathematical statements can be accepted as valid through axiomatic coherence, logical consistency, rule-governed proof procedures, and, in contemporary contexts, computationally checkable verification. Therefore, this study does not treat mathematical certainty as an absolute metaphysical truth, but as a formal and epistemic condition constructed within axiomatic, proof-theoretic, logical, and computational frameworks.

Recent developments in formal logic and proof theory also show that questions of mathematical certainty are increasingly connected to the regulation of inference, proof-theoretic validity, and the structural analysis of formal derivations (Golan, 2022). Recent studies in formal mathematics show that mathematical certainty is increasingly discussed through several interconnected domains: axiomatic foundations, proof-theoretic mechanisms, logical architectures, and computational verification. Studies on

axiomatic systems and formal foundations examine how mathematical theories are structured through explicitly stated assumptions, set-theoretic frameworks, arithmetic axiomatization, and the distinction between axiomatic and non-axiomatic mathematics (Khomskii & Oddsson, 2024; Lovyagin & Lovyagin, 2021; Salehi, 2022). These studies indicate that axioms function not merely as starting assumptions, but also as structural constraints that define the boundaries of valid mathematical reasoning.

At the proof-theoretic level, recent works examine how certainty is maintained through formal mechanisms such as cut-elimination, provability logic, sequent calculus, and structured systems of derivation (Gherardi et al., 2024; Gius & Tompits, 2023; Kushida, 2020). These mechanisms contribute to mathematical rigor by making derivations traceable, rule-governed, and internally checkable within a given formal system. Meanwhile, studies on modal, intuitionistic, constructive, lattice-based, and paraconsistent logics demonstrate that formal validity may vary across logical systems while remaining structurally regulated (Bílková et al., 2025; Chen et al., 2022; Miranda-Perea et al., 2022; von Plato, 2025). This suggests that mathematical certainty should not be reduced to a single logical framework, but can be examined as a system-relative condition shaped by the architecture of inference.

In addition, computational verification has become an important dimension in contemporary discussions of mathematical rigor. Machine-checked formalization and proof assistants such as Coq or Rocq allow mathematical proofs to be represented as formally verifiable objects (Dou & Yu, 2024; Guan et al., 2026). Such developments do not replace axiomatic reasoning or proof theory, but they strengthen the reproducibility and traceability of formal proofs by reducing dependence on informal human inspection.

Despite these developments, existing studies tend to examine axiomatic systems, proof theory, logical architectures, and computational verification separately or within specific theoretical domains. In addition, discussions of rigor and proof indicate that mathematical certainty requires conceptual clarification because proof, validity, and rigor are not always treated as identical notions in the literature (Zayton, 2022). As a result, the relationship among these components remains insufficiently synthesized in the literature. In particular, there is still limited systematic review evidence that explains how contemporary studies published between 2020 and 2026 conceptualize mathematical certainty across axiomatic, proof-theoretic, logical, and computational perspectives. This gap is important because the reviewed literature indicates that mathematical certainty is discussed not only in relation to foundational assumptions, but also in relation to the mechanisms through which formal reasoning is structured, validated, and verified.

To address this gap, this study conducts a systematic literature review of peer-reviewed studies retrieved from the Scopus database and published between 2020 and 2026. The review aims to synthesize how mathematical certainty is conceptualized in contemporary formal mathematical systems through four analytical dimensions: axiomatic structures, proof-theoretic mechanisms, logical architectures, and computational verification. Rather than proposing a universal theory of mathematical certainty, this study develops an interpretive conceptual framework based on thematic patterns identified in the selected literature.

Accordingly, this study is guided by the following research questions: a) How is mathematical certainty conceptualized in contemporary studies on axiomatic and formal systems?; b) How do proof-theoretic mechanisms contribute to the validation of mathematical reasoning within formal systems?; c) How do alternative logical architectures frame system-relative forms of validity and certainty?; d) What conceptual framework can be synthesized from the reviewed literature to explain the relationship among axiomatic foundations, proof mechanisms, logical architectures, and computational verification?

This study contributes to the literature in three main ways. First, it provides an updated systematic synthesis of contemporary studies published between 2020 and 2026 on mathematical certainty,

axiomatic systems, formal logic, proof theory, and computational verification. Second, it clarifies mathematical certainty as a structured and system-relative condition within formal mathematical systems, rather than as an undefined philosophical abstraction. Third, it proposes a layered conceptual framework that connects axiomatic foundations, proof-theoretic mechanisms, logical architectures, and computational verification as interrelated dimensions of contemporary mathematical rigor. Within the scope of the reviewed literature, this framework offers a conceptual lens for understanding how mathematical certainty may be constructed, maintained, and strengthened in contemporary formal mathematics.

## METHOD

This study employed a Systematic Literature Review (SLR) to examine how mathematical certainty is conceptualized within axiomatic systems, proof theory, formal logic, and computational verification. The review followed the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines, consisting of identification, screening, eligibility, and inclusion stages (Page et al., 2021). An SLR design was selected because this study aimed to synthesize conceptual and theoretical developments rather than to test an experimental intervention or aggregate empirical effect sizes.

The literature search was conducted in the Scopus database in February 2026. Scopus was selected because it provides broad coverage of peer-reviewed publications in mathematics, logic, philosophy of mathematics, and theoretical computer science. Although other databases may contain relevant studies, this review was limited to Scopus to ensure database consistency, indexing quality, and reproducibility. The search was conducted using the following search string:

TITLE-ABS-KEY ("axiomatic systems" OR "formal logic" OR "rigorous theory") AND PUBYEAR > 2019 AND PUBYEAR < 2026

The search produced 177 records. After removing 9 duplicate records, 168 records were screened based on titles and abstracts. During the screening stage, 121 records were excluded because they were not directly related to axiomatic systems, formal logic, proof theory, or mathematical certainty. In the eligibility stage, 47 full-text articles were assessed, and 27 articles were excluded because they did not meet the inclusion criteria or lacked explicit formal-theoretical discussion. Finally, 20 studies were included in the final synthesis.

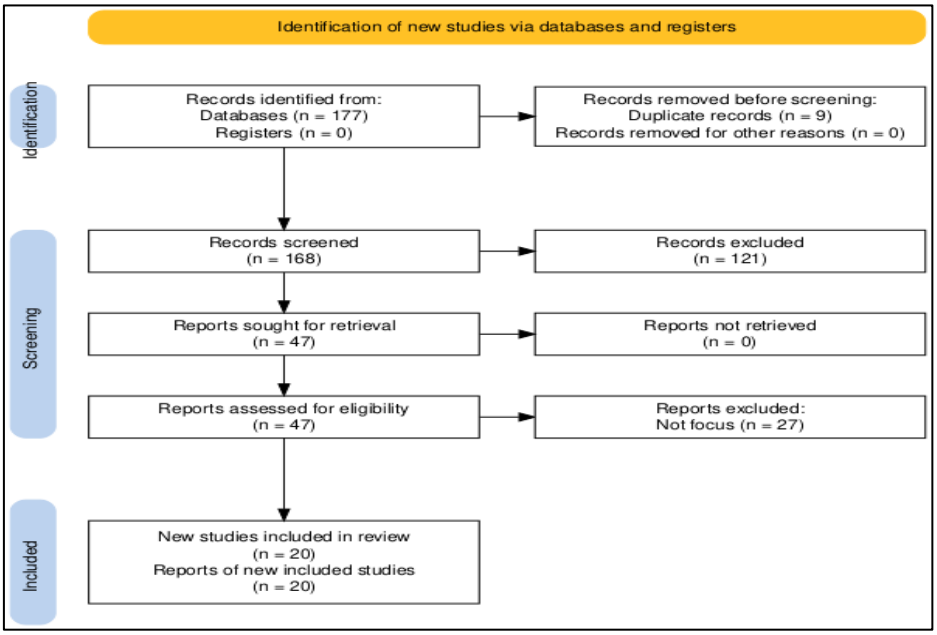


**Figure 1**. PRISMA flow diagram of the study selection process

**Table 1.** Inclusion and Exclusion Criteria

| Criteria | Inclusion | Exclusion |
|---|---|---|
| Publication year | Studies published from 2020 to 2026 | Studies published before 2020 |
| Source | Scopus-indexed peer-reviewed articles or conference papers | Non-indexed publications, editorials, and non-peer-reviewed sources |
| Topic relevance | Studies discussing axiomatic systems, formal logic, proof theory, formalization, theorem proving, or computational verification | Studies focused only on applied mathematics, education, engineering, or technology without formal-theoretical relevance |
| Conceptual relevance | Studies contributing to mathematical rigor, proof validity, formal reasoning, logical consistency, or mathematical certainty | Studies that do not discuss formal structures or logical reasoning |
| Accessibility | Full text available | Full text unavailable |

To strengthen the quality of the review, a quality appraisal procedure was conducted. Since the selected studies were mainly theoretical, logical, philosophical, and formalization studies, the appraisal did not use experimental indicators such as sample size, intervention validity, or statistical reliability. Instead, each study was assessed using four criteria: topical relevance, conceptual clarity, methodological transparency, and contribution to understanding mathematical rigor or certainty. Studies that met at least three of the four criteria were retained in the final corpus.

**Table 2.** Quality Appraisal Criteria

| Appraisal Aspect | Indicator |
|---|---|
| Topical relevance | Directly discusses axiomatic systems, formal logic, proof theory, formalization, theorem proving, or computational verification |
| Conceptual clarity | Provides explicit theoretical, logical, or formal contribution |
| Methodological transparency | Explains the formal, philosophical, proof-theoretic, or computational approach clearly |
| Contribution to the review | Contributes to understanding mathematical rigor, validity, proof correctness, logical consistency, or certainty |

Data were extracted using a structured extraction matrix. The extraction focused on conceptual contribution rather than statistical outcomes because the reviewed studies were not empirical intervention studies. Each article was analyzed based on the type of axiomatic or formal structure discussed, the logical framework employed, the proof-theoretic or verification approach used, and its contribution to mathematical rigor, validity, proof correctness, consistency, or certainty.

Data analysis was conducted using thematic conceptual synthesis. First, open coding was used to identify recurring concepts such as axiomatic coherence, formal constraints, proof normalization, derivational traceability, sequent calculus, modal validity, intuitionistic reasoning, paraconsistency, theorem proving, and machine-checked verification. Second, related codes were grouped into broader analytical categories. Initial coding produced three broad categories: axiomatic architecture, logical architecture, and rigor architecture. Through iterative refinement, these categories were reorganized into four thematic layers: axiomatic foundations, proof-theoretic mechanisms, logical architectures, and computational verification.

To improve trustworthiness, the coding results were repeatedly checked against the extracted data matrix. Theme boundaries were refined by ensuring that each theme was

supported by multiple studies in the corpus. The consistency of interpretation was strengthened through iterative comparison among the selected articles so that the final themes reflected recurring patterns in the literature rather than isolated claims from individual studies.

Because this review was conducted using a single database, the findings should be interpreted within the methodological scope of a Scopus-based corpus rather than as an exhaustive account of mathematical certainty. Therefore, the proposed framework is positioned as an interpretive synthesis based on the reviewed literature.

## RESULTS AND DISCUSSION

The thematic synthesis of the twenty selected studies produced four interrelated thematic layers: axiomatic foundations, proof-theoretic mechanisms, logical architectures, and computational verification. These themes were derived from recurring conceptual patterns across the reviewed literature rather than from predetermined categories. The selected studies did not treat mathematical certainty as a single and fixed philosophical guarantee, but as a condition associated with axiomatic coherence, formal constraint, derivational validity, system-relative inference, and reproducible proof validation.

### Axiomatic Foundations and Structural Coherence

The first thematic layer concerns axiomatic foundations and structural coherence. Several studies in the corpus examined set-theoretic foundations, axiomatic reformulations, arithmetic axiomatization, and the distinction between axiomatic and non-axiomatic mathematics. These studies include works on paraconsistent and paracomplete Zermelo-Fraenkel set theory, axiomatic set theory, hyperrational non-standard analysis, and philosophical analysis of axiomatic and non-axiomatic mathematics (Ilic & Višnjić, 2025; Khomskii & Oddsson, 2024; Lovyagin & Lovyagin, 2021; Salehi, 2022).

Across these studies, axioms are not presented merely as self-evident truths or metaphysical foundations. Rather, they function as formal constraints that define the boundaries of valid reasoning within a mathematical system. Axiomatic structures determine primitive assumptions, regulate admissible constructions, and provide the initial formal space in which proof-theoretic and logical operations can take place.

**Table 3.** Axiomatic Foundations and Structural Coherence

| Study Focus | Representative Studies | Structural Contribution |
|---|---|---|
| Set-theoretic reformulation | Khomskii & Oddsson (2024) | Shows that formal stability may be preserved under alternative logical regimes |
| Axiomatic verification of theorems | Zhang & Yu (2025) | Demonstrates the role of axioms as inferential boundary conditions |
| Arithmetic axiomatization | Lovyagin & Lovyagin (2021) | Indicates that formal certainty is system-relative and structurally constrained |
| Axiomatic and non-axiomatic mathematics | Salehi (2022) | Reinterprets axioms as structural conditions rather than absolute foundations |
| Formal foundation construction | Ilic & Višnjić (2025) | Suggests that formal systems require coherent structural organization |

This synthesis shows that axiomatic foundations contribute to mathematical certainty by establishing the structural conditions under which reasoning can be considered valid. However, the reviewed studies do not support the claim that axioms alone produce mathematical certainty. Rather, axiomatic systems provide the formal foundations upon which proof-theoretic

mechanisms, logical architectures, and verification procedures operate.

Findings: In the reviewed literature, axiomatic foundations support mathematical certainty by defining the structural constraints on valid reasoning within formal systems.

**Proof-Theoretic Mechanisms and Derivational Validity**

The second thematic layer concerns proof-theoretic mechanisms. A group of studies examined how mathematical rigor is maintained through formal proof structures, including cut-elimination, normalization, provability logic, sequent calculus, rejection systems, and specialized proof systems (Gherardi et al., 2024; Gius & Tompits, 2023; Hayashi, 2022; Miranda-Perea et al., 2022).

Across these studies, proof theory is consistently associated with the regulation of derivations. It clarifies how conclusions may be obtained from assumptions under explicitly defined rules. In this sense, proof-theoretic mechanisms strengthen mathematical certainty by making derivations traceable, reducible, and rule-governed. However, this should not be understood as proof theory independently guaranteeing mathematical truth. Its role is more precisely to validate derivations within a given formal system.

**Table 4.** Proof-Theoretic Mechanisms and Derivational Validity

| **Study Focus** | **Representative Studies** | **Structural Contribution** |
|---|---|---|
| Cut-elimination and normalization | Hayashi (2022) | Shows how derivational reduction supports proof stability |
| Provability logic | Kushida (2020) | Links proof structures to formal limits of truth and provability |
| Rejection systems | Gius & Tompits (2023) | Makes inferential steps explicit and traceable |
| Constructive modal proof systems | Miranda-Perea et al. (2022) | Shows how proof structures vary across constructive systems |
| Modal and conditional proof theory | Chen et al. (2022) | Extends proof-theoretic tools to non-classical systems |
| Specialized implication proof systems | Gherardi et al. (2024) | Shows that inferential rigor depends on explicitly defined proof rules |

The synthesis shows that proof-theoretic mechanisms contribute to mathematical certainty by ensuring that reasoning is formally controlled. Certainty, in this context, is not derived only from the acceptance of axioms, but also from the disciplined transformation of assumptions into conclusions through explicit proof procedures.

Finding: Proof-theoretic mechanisms support mathematical certainty by validating derivations through normalization, traceability, and rule-governed inference within formal systems.

**Logical Architectures and System-Relative Validity**

The third thematic layer concerns logical architectures. Several studies examined modal logic, intuitionistic logic, constructive modal logic, lattice-based modal logic, paraconsistent expansion, path-based modal semantics, and formal-language boundaries (Bílková et al., 2025; Drago, 2021; Figueira & Goren-Roig, 2025; Kelikli, 2024; Lyon & Ostropolski-Nalewaja, 2024; van der Berg et al., 2023).

The reviewed literature indicates that mathematical certainty is not always framed within a single classical logical system. Instead, different logical architectures establish different standards of validity, inference, and semantic interpretation. Modal, intuitionistic, constructive, lattice-based, and paraconsistent systems demonstrate that validity may be system-relative while still formally regulated.

**Table 5.** Logical Architectures and System-Relative Validity

| Study Focus | Representative Studies | Structural Contribution |
|---|---|---|
| Modal interpretation of intuitionistic logic | von Plato (2025) | Shows relations between modal and intuitionistic reasoning |
| Paraconsistent modal expansion | Bílková et al. (2025) | Demonstrates the possibility of extending logic under controlled formal conditions |
| Path-based modal semantics | Figueira & Goren-Roig (2025) | Shows the role of semantic structure in determining validity |
| Decidability in modal systems | Lyon & Ostropolski-Nalewaja (2024) | Identifies formal limits of inference within modal systems |
| Lattice-based modal calculi | van der Berg et al. (2023) | Develops alternative deductive architectures beyond classical logic |
| Intuitionistic reasoning | Drago (2021) | Demonstrates the plurality of inferential foundations |
| Formal-language boundaries | Kelikli (2024) | Clarifies the limits of certainty in relation to paradoxes and formal language |

The synthesis shows that logical architectures contribute to mathematical certainty by defining the inferential environment in which validity is interpreted. The existence of multiple formal systems does not imply the absence of rigor. Rather, each logical architecture constructs its own regulated space of inference. This provides a theoretical basis for interpreting mathematical certainty as system-relative, but not arbitrary.

Finding: Logical architectures support mathematical certainty by providing system-relative frameworks in which validity, inference, and semantic interpretation are formally regulated.

**Computational Verification and Reproducible Proof Validation**

The fourth thematic layer concerns computational verification. Studies in this group examined machine-checked formalization, proof assistants, and formal verification of mathematical theorems(Dou & Yu, 2024; Guan et al., 2026; C. Zhang & Yu, 2025). These works introduce a contemporary dimension of mathematical rigor in which formal proofs can be represented as machine-checkable objects.

Unlike traditional proof validation, which relies heavily on expert inspection, computational verification emphasizes reproducibility, traceability, and mechanical checking. Proof assistants such as Coq or Rocq allow formal derivations to be encoded and checked according to explicit logical rules. This does not mean that computational verification independently produces mathematical certainty. Rather, it strengthens confidence in formal reasoning by reducing ambiguity, detecting possible errors, and making proof validation reproducible.

**Table 6.** Computational Verification and Reproducible Proof Validation

| Study Focus | Representative Studies | Structural Contribution |
|---|---|---|
| Machine-checked formalization of classical theory | Guan et al. (2026) | Shows that classical theories can be represented in machine-checkable form |
| Mechanized proof validation | Dou & Yu (2024) | Presents mathematical proofs as traceable computational objects |
| Formal verification based on axiomatic set theory | Zhang & Yu (2025) | Demonstrates how derivations from axiomatic foundations can be checked mechanically |

The synthesis indicates that computational verification functions as a reinforcing layer of mathematical certainty. It does not replace axiomatic reasoning, proof theory, or logical analysis, but complements them by strengthening reproducibility and traceability in formal proof validation.

Finding: Computational verification supports mathematical certainty by strengthening reproducibility, traceability, and mechanical checkability in formal proof validation.

**Cross-Thematic Synthesis: The Layered Architecture of Mathematical Certainty**

The four thematic layers identified in this review were developed through iterative thematic synthesis. They were not imposed before the analysis, but emerged from repeated comparison of the selected studies and their contributions to the understanding of mathematical rigor and certainty. Axiomatic foundations appeared as the first layer because they define the formal constraints of reasoning. Proof-theoretic mechanisms were positioned as the second layer because they regulate derivations within those constraints. Logical architectures were placed as the third layer because they determine the broader inferential environment in which validity is interpreted. Computational verification was positioned as the fourth layer because it strengthens reproducibility through machine-checkable proof validation.

**Table 7.** Synthesis of the Layered Framework of Mathematical Certainty

| Layer | Function in the Framework | Contribution to Mathematical Certainty |
|---|---|---|
| Axiomatic foundations | Define primitive assumptions and formal constraints | Establish the structural boundaries of valid reasoning |
| Proof-theoretic mechanisms | Regulate derivations through rules, normalization, and traceability | Validate the movement from assumptions to conclusions |
| Logical architectures | Provide system-relative standards of inference and validity | Explain how different formal systems organize valid reasoning |
| Computational verification | Mechanically checks formal derivations | Strengthens reproducibility and reduces dependence on informal inspection |

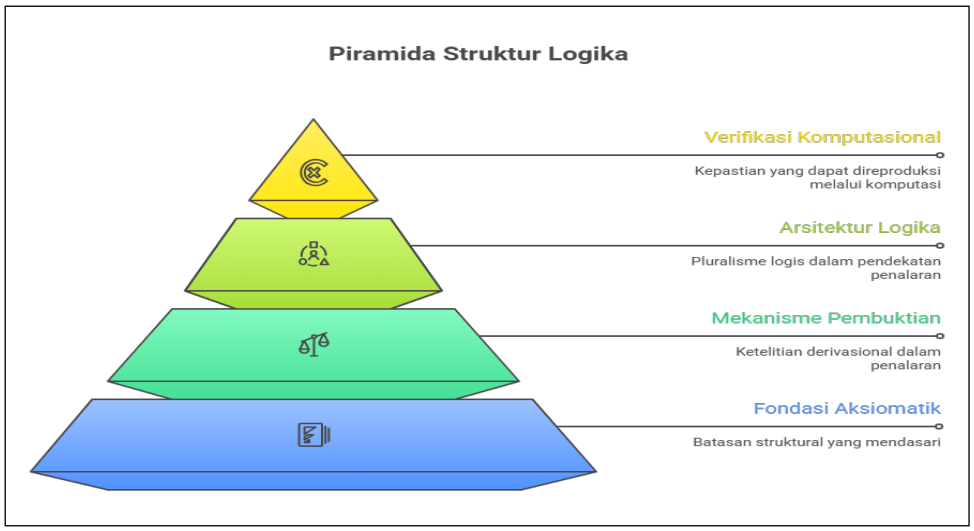


**Figure 2.** Layered Framework of Mathematical Certainty

This synthesis suggests that mathematical certainty, within the scope of the reviewed literature, may be interpreted as a layered and structurally mediated condition. It is not located in axioms alone, nor in proof mechanisms, logical systems, or computational verification in isolation. Rather, the selected studies indicate that mathematical certainty is supported by the interaction among formal constraints, derivational regulation, system-relative validity, and reproducible proof validation.

Finding: The reviewed literature supports an interpretive layered framework in which mathematical certainty is understood as a structurally mediated condition arising from the interaction of axiomatic, proof-theoretic, logical, and computational dimensions.

**Reinterpreting Mathematical Certainty as a Structural and System-Relative Condition**

The findings of this systematic literature review suggest that mathematical certainty in contemporary formal systems may be understood as a structural and system-relative condition rather than as a single absolute guarantee. Across the reviewed studies, certainty is supported by the interaction among axiomatic constraints, proof-theoretic regulation, logical architecture, and reproducible proof validation (Bílková et al., 2025; Guan et al., 2026; Hayashi, 2022; Khomskii & Oddsson, 2024).

At the axiomatic level, the reviewed literature indicates that axioms function as structural constraints that define the boundaries of valid reasoning. This finding refines the traditional view that mathematical certainty depends primarily on the acceptance of foundational assumptions. Axioms remain essential, but their role is not to produce certainty in isolation. Rather, they provide the formal space within which proof-theoretic mechanisms and logical architectures can operate (Khomskii & Oddsson, 2024; Liang et al., 2025; Lovyagin & Lovyagin, 2021; Salehi, 2022)

Proof-theoretic mechanisms further strengthen this structure by regulating how conclusions are derived from assumptions. The studies on cut-elimination, provability logic, sequent-type rejection systems, and specialized proof systems show that mathematical rigor depends on the traceability and rule-governed character of derivations (Gherardi et al., 2024; Gius & Tompits, 2023; Hayashi, 2022; Kushida, 2020). However, proof theory does not independently establish mathematical truth. Its contribution lies in validating derivations within a specified formal system.

The logical architecture layer adds another important dimension. The reviewed studies on modal, intuitionistic, constructive, lattice-based, and paraconsistent logics show that validity can be organized differently

across formal systems (Drago, 2021; Miranda-Perea et al., 2022; van der Berg et al., 2023; von Plato, 2025). This does not weaken mathematical rigor; rather, it shows that certainty may be system-relative while remaining formally constrained. In this sense, logical pluralism does not necessarily imply relativism, because each system maintains its own explicit rules of inference, semantic boundaries, and validity conditions.

Computational verification extends this layered structure by introducing mechanical checkability and reproducibility into formal proof validation. Machine-checked proofs and proof assistants do not replace axiomatic systems, proof theory, or logical frameworks. Instead, they reinforce them by reducing ambiguity, detecting possible errors, and making formal derivations more reproducible (Dou & Yu, 2024; Guan et al., 2026; C. Zhang & Yu, 2025) Thus, computational verification should be interpreted as a strengthening mechanism rather than as an independent source of mathematical certainty.

**Relation to Formalism, Logicism, Structuralism, and Intuitionism**

The proposed layered framework can be positioned in relation to major philosophical perspectives in mathematics. First, in relation to formalism, the findings support the idea that mathematical certainty depends on symbolic systems governed by explicit rules. The role of axioms, inference rules, proof systems, and formal verification in the reviewed literature is consistent with the formalist emphasis on rule-governed manipulation within formal systems (Gherardi et al., 2024; Ilic & Višnjić, 2025; Salehi, 2022). However, the framework extends this view by showing that certainty is not only supported by formal rules, but also strengthened through proof-theoretic transparency, logical architecture, and computational reproducibility.

Second, in relation to logicism, the findings confirm that logic remains central to the construction of mathematical certainty. Studies on modal logic, intuitionistic logic, paraconsistent logic, and proof theory demonstrate that formal reasoning depends on logical structures (Bílková et al., 2025; Kushida, 2020; von Plato, 2025). However, the reviewed literature does not support a single universal logical foundation. Instead, it suggests that different logical systems may generate different but internally regulated standards of validity. Therefore, the layered framework modifies the logicist expectation of a unified logical foundation by emphasizing logical plurality within formal constraint.

Third, the findings are strongly aligned with structuralism. In the reviewed literature, mathematical certainty appears less dependent on the intrinsic nature of mathematical objects and more dependent on the coherence of relations, rules, and structures within formal systems. Axioms define structural boundaries, proof theory regulates derivations, logical architectures organize validity, and computational verification checks reproducibility (Ilic & Višnjić, 2025; Khomskii & Oddsson, 2024; Salehi, 2022; C. Zhang & Yu, 2025). This supports a structural interpretation of certainty as something maintained through the organization of formal relations rather than through isolated foundational assumptions.

Fourth, the framework also relates to intuitionism, particularly through studies on intuitionistic logic and constructive modal logic. From this perspective, certainty is connected to the construction and justification of proofs rather than to abstract truth conditions alone. The reviewed studies show that constructive and intuitionistic systems provide alternative ways of understanding validity, where proof construction and epistemic accessibility become central (Drago, 2021; Miranda-Perea et al., 2022; von Plato, 2025). This supports the idea that mathematical certainty can be interpreted differently depending on the underlying logical architecture.

Taken together, these comparisons show that the proposed framework does not simply repeat existing philosophical positions. Rather, it integrates elements from formalism, logicism, structuralism, and intuitionism into a layered account of contemporary mathematical certainty. The framework is formalist in its attention to rules, logicist in its attention to inferential

systems, structuralist in its emphasis on relational coherence, and intuitionistic in its recognition of proof construction and system-specific validity.

**Theoretical Contribution of the Layered Framework**

The main theoretical contribution of this review is the formulation of a layered framework for interpreting mathematical certainty in contemporary formal systems. This framework does not claim to provide a universal theory of mathematical certainty. Instead, it offers an interpretive synthesis based on recurring patterns identified across the twenty selected studies.

The framework contributes to the literature in three ways. First, it clarifies that mathematical certainty should not be reduced to axiomatic consistency alone. Although axiomatic coherence remains fundamental, the reviewed literature shows that certainty also depends on proof-theoretic regulation, logical architecture, and verification mechanisms (Bílková et al., 2025; Guan et al., 2026; Hayashi, 2022; Khomskii & Oddsson, 2024).

Second, the framework explains how different formal systems can support different forms of validity without abandoning rigor, as shown in studies on modal, intuitionistic, constructive, lattice-based, and paraconsistent systems (Drago, 2021; Miranda-Perea et al., 2022; van der Berg et al., 2023; von Plato, 2025).

Third, it highlights the growing importance of computational verification in contemporary mathematics, not as a replacement for human reasoning, but as a mechanism for strengthening formal proof validation (Dou & Yu, 2024; Guan et al., 2026; C. Zhang & Yu, 2025).

This layered interpretation is useful because it connects research domains that are often discussed separately. Studies on axiomatic systems, proof theory, non-classical logic, and computational verification are frequently treated as distinct areas. The present synthesis suggests that these areas can be understood as interrelated dimensions in the construction and stabilization of mathematical certainty.

**Implications for Contemporary Formal Mathematics**

The findings have several implications for contemporary formal mathematics. First, they suggest that the role of axioms should be understood more dynamically. Axioms are not merely initial assumptions; they function as formal constraints that organize the space of valid reasoning. This means that certainty depends not only on what is assumed, but also on how those assumptions regulate inference (Khomskii & Oddsson, 2024; Lovyagin & Lovyagin, 2021; Salehi, 2022).

Second, the findings highlight the importance of proof-theoretic transparency. In complex formal systems, mathematical rigor depends on whether derivations can be traced, reduced, and validated according to explicit rules. This reinforces the importance of proof systems, sequent calculi, normalization procedures, and related mechanisms in maintaining formal rigor (Gherardi et al., 2024; Gius & Tompits, 2023; Hayashi, 2022; Kushida, 2020).

Third, the findings indicate that logical pluralism is not necessarily a threat to mathematical certainty. Different logical systems may define different validity conditions, but they can still maintain rigor if their inferential rules and semantic constraints are explicitly regulated. This is especially relevant in contemporary studies of modal, intuitionistic, constructive, lattice-based, and paraconsistent systems (Bílková et al., 2025; Figueira & Goren-Roig, 2025; Lyon & Ostropolski-Nalewaja, 2024; van der Berg et al., 2023; von Plato, 2025).

Fourth, computational verification introduces a new level of reproducibility into mathematical practice. Machine-assisted proofs allow formal derivations to be checked systematically and repeatedly. This strengthens confidence in proof validation, especially in complex formalizations where informal inspection may be insufficient (Dou & Yu, 2024; Guan et al., 2026; C. Zhang & Yu, 2025).

**Limitations and Future Directions**

This review has several limitations. First, the corpus was limited to twenty studies retrieved from the Scopus database. Although Scopus provides broad coverage of peer-

reviewed literature, relevant studies from other databases may not have been included. Therefore, the findings should be interpreted within the scope of a Scopus-based systematic literature review.

Second, this study focused on conceptual and theoretical contributions rather than empirical outcomes. As a result, the review did not evaluate mathematical certainty through experimental data, statistical evidence, or classroom implementation. This is consistent with the purpose of the article as a conceptual systematic literature review, but it also limits the generalizability of the proposed framework.

Third, the four-layer framework should be understood as an interpretive synthesis rather than a final or exhaustive model. Future research could expand the corpus by including studies from Web of Science, MathSciNet, PhilPapers, or other relevant databases. Further studies could also examine how the proposed framework applies to specific areas such as automated theorem proving, formalized mathematics, constructive mathematics, or paraconsistent foundations.

Overall, the reviewed literature suggests that mathematical certainty in contemporary formal systems is best understood as a layered and structurally mediated condition. It is supported by axiomatic coherence, proof-theoretic regulation, logical architecture, and computational reproducibility. This perspective provides a more integrated understanding of how mathematical rigor is constructed, maintained, and strengthened in contemporary formal mathematics.

## CONCLUSION

This systematic literature review examined how mathematical certainty is conceptualized in contemporary studies on axiomatic systems, proof theory, logical architectures, and computational verification. Based on twenty Scopus-indexed studies published between 2020 and 2026, the reviewed literature suggests that mathematical certainty is not treated as a single metaphysical guarantee, but as a structurally mediated condition within formal mathematical systems.

The findings answer the research questions by showing that mathematical certainty is supported by four interrelated dimensions. Axiomatic foundations define the structural boundaries of valid reasoning; proof-theoretic mechanisms validate derivations through normalization, traceability, and rule-governed inference; logical architectures provide system-relative standards of validity; and computational verification strengthens reproducibility and mechanical checkability in formal proof validation.

The main theoretical contribution of this review is the formulation of a layered conceptual framework for interpreting mathematical certainty in contemporary formal systems. This framework connects axiomatic coherence, proof-theoretic regulation, logical plurality, and computational reproducibility as interrelated dimensions of mathematical rigor. However, the framework should be understood as an interpretive synthesis based on the reviewed literature, not as a universal or exhaustive theory of mathematical certainty.

This study is limited to twenty studies retrieved from the Scopus database; therefore, relevant works from other databases may not have been included. Future research may expand the corpus by including additional databases such as Web of Science, MathSciNet, PhilPapers, or IEEE Xplore, and may examine the application of the proposed framework in automated theorem proving, formalized mathematics, constructive logic, paraconsistent foundations, and machine-assisted proof systems.

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11

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